Poster Abstract: A Game Theoretic Approach for Slotted Medium Access in Wireless Networks

Hua Liu and Bhaskar Krishnamachari
Departments of Computer Science and Electrical Engineering
Viterbi School of Engineering
University of Southern California
Los Angeles, CA 90089

I. Introduction

In practice, terminals in wireless networks usually belong to different individual users. Therefore, these terminals inherently care more about their own benefit than the overall system benefit. This kind of selfishness is especially emphasized in medium access control, where each transmitting user in the system wants to let his/her packet get through to the respective receiver as early as possible, while incurring minimum energy expenditure. Game theory, originally used in economics to model human interactions, is a powerful tool to analyze and solve self-centric problems in distributed systems. It is attracting more and more attention from the wireless networking research community in recent years.

In game theory, each selfish player's goal is to choose a possible strategy or a probabilistic mixture of strategies in the strategy space to maximize his/her own utility function [1]. The Nash equilibrium is an important concept in game theory for both pure strategy and mixed strategy games. A Nash equilibrium is a set of pure/mixed strategies, one for each player, such that no player has incentive to unilaterally deviate from this set.

II. Problem Formulation

We first describe the assumptions underlying our medium access game problem formulation. We assume that there are N player nodes contending for access on a common medium (e.g., trying to transmit to the same receiver). The players all have the same priority, the same workload and the same remaining power at the beginning. In the one-shot game we discuss here, each node has only one packet to transmit in a single frame consisting of K slots (this is a key difference from prior game theoretic treatments of slotted Aloha [2]). A node's transmission in a given slot is successful if and only if all other nodes do not transmit in that slot. We also assume that the nodes are selfish but rational; a node that has transmitted successfully in one slot will not transmit in later slots.

A. The Utility Function

The utility function is an important component for a game theoretic formulation. It expresses how much a player gains by taking a certain action. For our slotted medium access problem, we consider two major factors: time delay and power consumption.

In every slot, each player has two possible actions: transmit (T) or wait (W). We define the transmission cost for player i to be c_i and waiting cost to be 0. In practice, c_i can represent the power consumption of transmission. If the transmission fails (due to a collision with another player), player i's benefit is 0. If the transmission succeeds in slot k, player i will get benefit of $P\delta^{k-1}$, where P denotes the benefit value from successful transmission in slot 1 and δ is defined to be a decay parameter whose value is in $(0,1)$. This δ parameter is related to delay as it gives players an incentive to finish transmission as early as possible. Without loss of generality, we set P to be 1 and use normalized transmission cost c_i. Let $U(i,k)$ denote the utility for player i in slot k. Then we have the following utility function:

$$U(i,k) = \begin{cases} 0 & Wait \\ -c_i & Failed\ transmission \\ P\delta^{k-1} - c_i & Successful\ transmission \end{cases} \quad (1)$$

In order to let each player have enough incentive to transmit even in the last slot of a frame, we let $P \times \delta^{K-1} > c_i$, where K denotes total number of slots in our one-shot game. The strategy space for player i can be expressed as a row vector of player i's transmission probability $p_{i,k}$ in slot k. Let l_i denote the strategy space of player i, then we have $l_i = (p_{i,1}, p_{i,2}, ..., p_{i,K})$.

B. The Global Optimal Formulation

In our first approach we look at the global optimum cooperative solution for this problem, which maximizes the utility for all players. We force all players to take symmetric strategies, which provides fairness when they all have the same payoff function.

The following is the expression of expected payoff for player i over all the slots in the game:

$$E(U_i) = \sum_{j_1, j_2, \ldots, j_K} [(\prod_{k=1}^{K} \binom{N}{j_k} p_k^{j_k} \times (1-p_k)^{N-j_k}) \times \sum_{k=1}^{K} U(i,k)] - M(U_i) \quad (2)$$

In Equation 2, $j_k = 0, 1, 2, ..., N$ for all $k = 1, 2, 3..., K$. $E(U_i)$ denotes the expected payoff for player i. N represents the total number of players in the game. p_k denotes the transmit probability in slot k. Since the strategy is symmetric, we can simplify the notation $p_{i,k}$ to p_k. $U(i,k)$ denotes the utility for player i in slot k with the corresponding strategy. The function $M(U_i)$ denotes the invalid combinations in the strategy space. For example, if one player successfully transmitted in a previous slot, he cannot transmit again in all the following slots. The expression for $M(U_i)$ is omitted here for brevity.

Our objective for the global optimization formulation is to maximize the value of $E(U_i)$ in a K-dimension space. That is,

$$Max\ E(U_i)$$
$$subject\ to\ \ 0 \le p_k \le 1\ for\ k = 1, 2, ..., K$$

C. The Mixed Strategy Equilibrium

While the formulation of the previous section is straightforward, that global optimum cannot be attained in the presence of selfish users that would cheat to maximize their own benefit. To characterize the selfishness of players in the system, a mixed strategy equilibrium solution is necessary.

According to game theory, at an equilibrium point, no player has the incentive to unilaterally change his action. A mixed strategy game always has a Nash Equilibrium. Similar to Section II-B, the expected payoff over all the slots for player i is presented in Equation 3.

$$E(U_i) = \sum_{m_{j,k}} [(\prod_{k=1}^{K} \prod_{j=1}^{N} p_{j,k}^{m_{j,k}} \times (1-p_{j,k})^{1-m_{j,k}}) \times \sum_{k=1}^{K} U(i,k)] - M(U_i) \quad (3)$$

In this equation, $m_{j,k} = 0, 1$ for all the (j,k) tuples. In slot k, $m_{j,k} = 0$ implies that player j acts waiting and $m_{j,k} = 1$ implies that player j transmits. $U(i,k)$ denotes the payoff for player i in slot k considering the current actions of each player. $M(U_i)$ denotes the sum of the invalid combinations, as before.

In order to get the equilibrium solution, we take partial differential of each $p_{i,k}$ with $E(U_i)$ and get a set of equations:

$$\frac{\partial E(U_i)}{\partial p_{i,k}} = 0,\ \forall i, k \quad (4)$$

D. Illustration using a simple 2×2 Game

In order to demonstrate the usage of both formulations, we show an example of a 2×2 game in this section. The game has two players who are trying to finish transmission in two time slots. The payoffs for each player with different strategies are listed in table II-D.

For this 2×2 game, there are $2^2 \times 2^2 = 16$ possible actions. Since each player only has one packet to transmit in each frame, there are $2 \times 2 = 4$ invalid combinations. We calculate the expected payoff for player 1 over all the remaining 12 different strategy combinations.

We plot the expected payoff value of two slots as a function of p_1 and p_2 in Fig 1. We can see that the global optimal

players in slot 1	Transmit	Wait
Transmit	(-1,-1)	(0,1)
Wait	(1,0)	(0,0)
players in slot 2	Transmit	Wait
Transmit	(-1,-1)	(0,0.5)
Wait	(0.5,0)	(0,0)

TABLE I

PAYOFF VALUES OF EACH STRATEGY FOR TWO PLAYERS

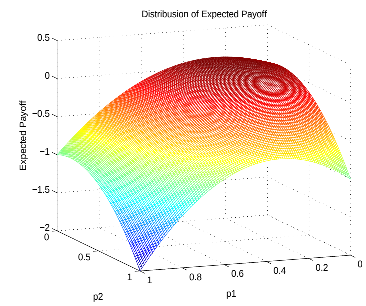


Fig. 1. Expected Payoff for 2×2 game with symmetric strategy constrains

expected payoff for each player is $E = 0.1372$ when $p_1 = 0.25$ and $p_2 = 0.11$. Note that both players have the same expected payoff in this illustration since they have identical payoff functions and employ symmetric strategies.

For the mixed strategy equilibrium solution, according to the definition in equation 4, we take derivatives of the expected payoff expression by $p_{1,1}$ and $p_{1,2}$ separately and set the partial differential to zero. We do the same operation on player 2. The resultant equations yield the following solution: $p_{1,1} = p_{2,1} = 0.4428$ and $p_{1,2} = p_{2,2} = 0.1471$. The expected payoff for each player is $E = 0.0507$. Although the strategies were not predetermined to be symmetric in this non-cooperative game (unlike in the global optimum solution), the symmetric payoff functions for the two players results in a symmetric strategy set at equilibrium.

III. CONCLUSIONS AND ONGOING WORK

We notice that the cooperative global optimization formulation provides a fair *Pareto* optimal solution that provides a reward for each player that is more than twice compared to that obtained in the non-cooperative game with selfish users. This is the motivation for our ongoing work on developing a mechanism with a more efficient equilibrium point. We are also working on computationally tractable approaches for identifying the global optimum and the non-cooperative equilibrium point in the case of large numbers of users and slots per frame. In the future, we plan to relax many of the idealized assumptions in the formulation and consider multi-hop settings, so that the results can be applied to real-world medium access standards for wireless ad-hoc networks.

REFERENCES

[1] D. Fundenberg and J. Tirole. In *Game theory*, 1991.
[2] A. B. MacKenzie and S. B. Wicker. Selfish users in aloha: A game-theoretic approach. In *IEEE Vehicular Technology Conference*, October 2001.

USC Autonomous Networks Research Group

A Game Theoretic Approach for Slotted Medium Access in Wireless Networks

Hua Liu and Bhaskar Krishnamachari
(hual@usc.edu, bkrishna@usc.edu)
Departments of Computer Science and Electrical Engineering,
Viterbi School of Engineering, University of Southern California

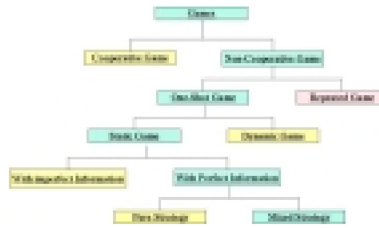
Introduction: Selfishness, Game Theory and Networking

Why Selfishness is important?

- Terminals in wireless networks usually belong to different individual users
- Reasonable to assume each individual only cares about self-gain from the system

Why Game Theoretic Approach?

- First applied in economics as a powerful tool to simulate behavior of humans
- Even better tool for analyzing and designing networks with distributed selfish entities
- Game theory states that a system will converge to equilibrium if there exists one
- Characterizing and improving the efficiency of this equilibrium is of practical interest



Taxonomy of Game Theory
The green path indicates the focus of our work

Problem Formulation: Global Optimization and Mixed Strategy Equilibrium

The Slotted Medium Access Game

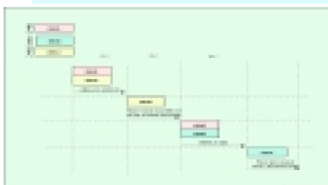


Illustration of The Game

Assumptions:

- Initially, each node is rational, has the same workload, and the same priority
- Each player has exactly one packet to transmit in a frame consisting of multiple slots (different from prior work [1])
- All nodes share the same channel. Successful transmission in a slot happens when only one player transmits

The Utility Function

$$U(i,k) = \begin{cases} 0 & Waiting \\ -c_i & Transmitting\ but\ fail \\ P \times \delta^{k-1} - c_i & Transmitting\ and\ succeed \end{cases}$$

The Utility Function

- Each node can either transmit or wait in each slot
- Considers two major parameters: delay and power consumption
- Delay is represented by δ and power consumption is denoted by c_i

A Player's Expected Payoff Over all Slots

$$E(U_i) = \sum_{j_1, j_2, \ldots, j_K} \left[\prod_{k=1}^{K} \binom{N}{j_k} p_k^{j_k} \times (1-p_k)^{N-j_k} \times \sum_{k=1}^{K} U(i,k) \right] - M(U_i)$$

The expected Payoff Function

- p_k is the transmit probability in slot k
- K is the total number of slots
- N is the total number of players
- $j_k = 0, 1, 2, 3, \ldots, N$ for $k = 1, 2, 3, \ldots K$

- We let the players have symmetric strategies to provide fairness
- $M(U_i)$ represents the illegal combinations of transmitting and waiting: a player will not transmit again after having successfully transmitted

Global Optimization Formulation

$$Max\ E(U_i)$$
$$subject\ to\ \ 0 \le p_k \le 1\ for\ k = 1, 2, \ldots, K$$

- Global optimization formulation provides optimal payoff for each player with consideration of fairness
- Cooperation needs to be enforced to achieve the optimal solution
- When taking selfishness into consideration, the system leads to a (generally inferior) Nash equilibrium point instead of this Pareto Optimal solution

Mixed Strategy Equilibrium

$$E(U_i) = \sum_{m_{j,k}} \left[\prod_{k=1}^{K} \prod_{j=1}^{N} p_{j,k}^{m_{j,k}} \times (1-p_{j,k})^{1-m_{j,k}} \times \sum_{k=1}^{K} U(i,k) \right] - M(U_i)$$

$m_{j,k} = 0,1$ for all (j, k). $m_{j,k}=0$ implies player j wait in slot k. $m_{j,k}=1$ implies player j transmits in slot k

Solve the following equations to get equilibrium point:

$$\frac{\partial E(U_i)}{\partial p_{i,k}} = 0,\ \forall i, k$$

An Example for Proposed Solution: A Simple Game where 2 Players Compete over 2 Slots

The Payoff Table

In Slot 1		Player 2	
		Transmit	Wait
Player 1	Transmit	(-1, -1)	(1, 0)
	Wait	(0, 1)	(0, 0)

In Slot 2		Player 2	
		Transmit	Wait
Player 1	Transmit	(-1, -1)	(0.5, 0)
	Wait	(0, 0.5)	(0, 0)

The Payoff Table for a 2x2 game

The Calculation

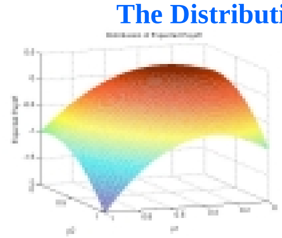
$$E = 3/2 * p_1^2 * p_2 - 2 * p_1^2 - 3/2 * p_1 * p_2 + 5/2 * p_1 * p_2^2$$
$$+ p_1 - 5/2 * p_1^2 * p_2^2 - 3/2 * p_2^2 + 1/2 * p_2$$

- There are totally 16 transmit/wait combinations, 4 of them are invalid. E is the sum of the remaining 12 valid combinations

$$E = (-2) \times p_{1,1} \times p_{2,1} + p_{2,1} \times p_{1,2} \times p_{2,2} + 3/2 \times p_{1,1} \times p_{2,2} \times p_{1,2}$$
$$-3/2 \times p_{1,1} \times p_{1,2} + 3/2 \times p_{1,1} \times p_{1,2} \times p_{2,1}$$
$$-5/2 \times p_{1,1} \times p_{1,2} \times p_{2,1} \times p_{2,2} - 3/2 \times p_{1,2} \times p_{2,2} + 1/2 \times p_{1,2}$$

- According to the symmetric characteristic of the payoff table, $p_{1,1}=p_{2,1}$ and $p_{1,2}=p_{2,2}$
- If the payoff table is asymmetric, it is not necessary to have symmetric solution

The Distribution of the Expected Payoff



The Solutions

Global Optimal Solution	Mixed Strategy Solution
$p_1 = 0.25$	$p_{1,1} = p_{2,1} = 0.4428$
$p_2 = 0.11$	$p_{1,2} = p_{2,2} = 0.1471$
$E = 0.1373$	$E(U_1) = E(U_2) = 0.0807$

- Global optimal solution yields better expected payoff for both players
- The system will automatically converge to mixed strategy solution if there is no policy constraint for selfish users in the system

Conclusions and Ongoing Work

- **Game theory has a promising future in solving practical medium access problems in wireless networks**
- **The ongoing work is to simplify the computational complexity of the formulation, and extend the framework for multi-hop scenarios**
- **We are also investigating a repeated game framework that uses the mixed strategy equilibrium to enforce cooperation among selfish nodes**

Related Work: [1] A. B. MacKenzie and S. B. Wicker, "Selfish Users in Aloha: A Game-Theoretic Approach," IEEE Vehicular Technology Conference, October 7-11, 2001